Mr. Michael T. Davis Calculus	Unit 1 Chapter 1 Practice Test September 26, 2016

Name: Solutions

1. Write the point-slope equation of the line containing the point $(2,3)$ and having slope $\frac{-3}{2}$

$y-3=\frac{-3}{2}(x-2)$

2. Write an equation of the line containing the points $(5,1)$ & $(5,8)$

$x=5$

3. A line has equation $5x-2y=-20$. Write a slope-intercept equation that contains the point $(10,1)$ and is perpendicular to the first line.

$-2y=-5x-20$

$y=\frac{5}{2}x+10$

$y-1=\frac{-2}{5}(x-10)$

$y=\frac{-2}{5}x+4+1$

$y=\frac{-2}{5}x+5$

4. Determine the two coordinates of the x-intercept of the line with equation $y=\frac{1}{3}x-4$

$y=0$ $\quad 0=\frac{1}{3}x-4$ $\quad 4=\frac{1}{3}x$ $\quad 12=x$ $\quad (x,y)=(12,0)$

5. Write the height h of an equilateral triangle as a function of its side length x. Then determine the heath of an equilateral triangle with a side length of 8 inches.

$h(x)=\frac{1}{2}x\cdot\frac{\sqrt{3}}{2}x=\frac{\sqrt{3}}{4}x^2$

6. Identify the domain and range of the function $f(x)=\sqrt{3-x}$

D: $x\leq 3$ $\quad$ R: $y\geq 0$

7. Identify the domain and range of the function $p(x)=x^2-4x$

D: $(-\infty,\infty)$ $\quad$ R: $y\geq -4$

8. Identify the domain and range of the function $w(x)=\frac{1}{x^2+4}$

D: $(-\infty,\infty)$ $\quad$ R: $(0,\frac{1}{4}]$

9. Identify the domain and range of the function $g(x)=\sqrt{16-x^2}$

D: $[-4,4]$ $\quad$ R: $[0,4]$

10. Identify the domain and range of the function $k(x)=2\sin x$

D: $(-\infty,\infty)$ $\quad$ R: $[-2,2]$

11. Identify the domain and range of the function $T(x)=2\tan x$

D: $\mathbb{R}$, $x\neq\frac{(2x+1)\pi}{2}$ $\quad$ R: $(-\infty,\infty)$

$4 - x^2 > 0$

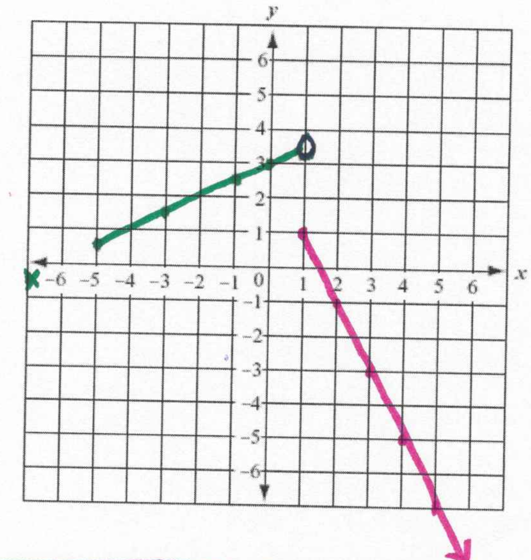
$4 - x^2 \neq 0$ $\quad 4 \neq x^2$

$x \neq \pm 2$

12. Identify the domain and range of the function $f(x) = \frac{1}{\sqrt{4-x^2}}$

D: $-2 < x < 2$ $\qquad$ R: $[\frac{1}{2}, \infty)$

13. Given the piecewise function $f(x) = \begin{cases} \frac{1}{2}x + 3 \text{ if } -5 \le x < 1 \\ -2x + 3 \text{ if } x \ge 1 \end{cases}$,



determine the following:

a. A graph the function

b. The value of $f(3)$ $= -3$

c. If the function is continuous at $x = 1$ $\quad$ NO

d. The domain of the function $\quad$ $-5 \le x$ or $[-5, \infty)$

14. Page 19 # 41 $\quad f(x) = \begin{cases} x \text{ if } 0 \le x \le 1 \\ -x + 2 \text{ if } 1 < x \le 2 \end{cases}$

15. Page 19 # 44 $\quad f(x) = \begin{cases} -3x - 3 \text{ if } -1 < x \le 0 \\ -2x + 3 \text{ if } 0 < x \le 2 \end{cases}$

16. Determine simplified expression for $f(g(x))$ and the domain of $f(g(x))$ given the functions $f(x) = 1 - x^2$ and $g(x) = \sqrt{x-2}$ $\quad f(\sqrt{x-2}) = 1 - (\sqrt{x-2})^2 = 1 - x + 2 = 3 - x$

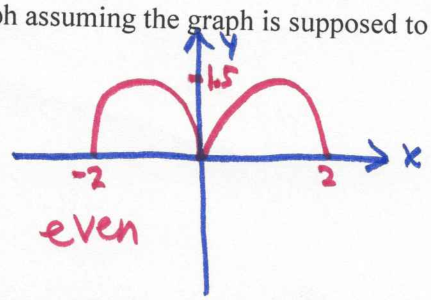
Domain comes from $x - 2 \ge 0$ $\quad x \ge 2$ $\quad$ D: $[2, \infty)$

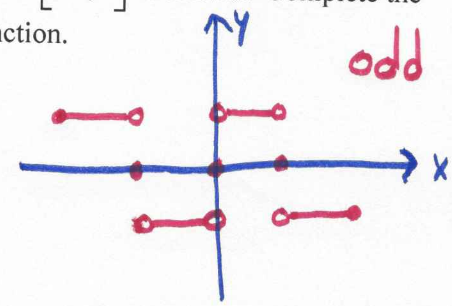
17. Determine simplified expression for $g(f(x))$ given the functions $f(x) = x^3 - 8$ and $g(x) = (x+8)^{\frac{1}{3}}$ $\quad g(x^3 - 8) = (x^3 - 8 + 8)^{\frac{1}{3}} = (x^3)^{\frac{1}{3}} = x$

Pg. 21 #67 18. A portion of a graph of a function defined on the interval $[-2, 2]$ is shown. Complete the graph assuming the graph is supposed to be an even function.

Pg. 21 #68 19. A portion of a graph of a function defined on the interval $[-2, 2]$ is shown. Complete the graph assuming the graph is supposed to be an odd function.

18.





20. Determine the zero of the function $f(x)=5-8^x$. Give the zero accurate to three decimal places.

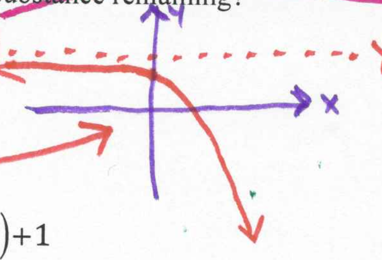
Solve $0=5-8^x$ $\quad 5=8^x$ $\quad \ln 5=\ln 8^x$ $\quad \ln 5=x\ln 8$ $\quad \frac{\ln 5}{\ln 8}=x$ $\quad x\approx 0.774$

21. Suppose the half-life of a certain radioactive substance is 20 days and there are 5 grams present initially. When will there be only 3 grams of the substance remaining?

$A(t)=5\left(\frac{1}{2}\right)^{t/20}$ $\quad 3=5\left(\frac{1}{2}\right)^{t/20}$ $\quad \frac{3}{5}=\left(\frac{1}{2}\right)^{t/20}$ $\quad \ln\left(\frac{3}{5}\right)=\frac{t}{20}\ln\left(\frac{1}{2}\right)$ $\quad t=\frac{20\ln\left(\frac{3}{5}\right)}{\ln\left(\frac{1}{2}\right)}$ $\quad t\approx 14.74$ days

22. What is the range of the function $f(x)=-2^x+3$

$(-\infty,3)$



23. What is the range of the function $f(x)=\ln(x-2)+1$

$(-\infty,\infty)$

24. Determine to three decimal places of accuracy how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded monthly.

$A(t)=A_0\left(1+\frac{0.0625}{12}\right)^{12t}$ $\quad t=$ years $\quad 2=1\left(1+\frac{0.0625}{12}\right)^{12t}$ $\quad \ln 2=12t\ln\left(1+\frac{0.0625}{12}\right)$ $\quad t=\frac{\ln 2}{12\ln\left(1+\frac{0.0625}{12}\right)}$ $\quad t\approx 11.12$ years

25. Determine to three decimal places of accuracy how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded continuously.

$A(t)=A_0e^{0.0625t}$ $\quad 2=1\cdot e^{0.0625t}$ $\quad \ln 2=0.0625t$ $\quad t=\frac{\ln 2}{0.0625}\approx 11.1$ years

26. The number of bacteria in a petri dish culture after t hours is given by the function $B(t)=100e^{0.693t}$

a. What was the initial number of bacteria present?

100 bacteria

b. How many bacteria are present after 6 hours?

$B(6)=100e^{0.693(6)}\approx 6{,}394$

c. Approximately when will the number of bacteria be 350?

$350=100e^{0.693t}$ $\quad 3.5=e^{0.693t}$ $\quad \ln 3.5=0.693t$ $\quad t\approx 1.81$ hours

27. What is the inverse function of $f(x)=2^x$?

$f^{-1}(x)=\log_2 x$

28. If $g(x)=e^{x-2}$, then determine $g^{-1}(x)$.

$y=e^{x-2}$ $\quad \ln y=\ln\left(e^{x-2}\right)$ $\quad \ln y=x-2$ $\quad \ln x=y-2$ $\quad y=\ln(x)+2$

29. If $k(x)=\ln(x-5)$, then determine $k^{-1}(x)$.

$y=\ln(x-5)$ $\quad x=\ln(y-5)$ $\quad x=\log_e(y-5)$ $\quad e^x=y-5$ $\quad y=e^x+5$ $\quad k^{-1}(x)=e^x+5$

30. Determine if the function $f(x)=\sqrt{x+6}$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

yes, because for each y value there corresponds exactly one x value

31. Determine if the function $g(x)=x^2-1$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

No, because there are y values that correspond with two x values, e.g. (2,3) & (-2,3).

32. Determine if the function $p(x)=\frac{1}{x-3}$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

Yes, because for each y there corresponds exactly one x.

33. Determine if the function $A(x)=\tan x$ is a one-to-one function. Answer with "yes" or "no" and explain why ore why not.

No, because there are many y values that correspond with multiple x values, i.e. $(\pi,0)$ & $(2\pi,0)$ & $(3\pi,0)$

34. A 1-to-1 function $y=f(x)$ is such that $f\left(\frac{1}{2}\right)=-4$. Determine the value of $f^{-1}(-4)$ $=\frac{1}{2}$

35. Solve the equation $\ln y=t+1$ for y. $y=e^{t+1}$

36. Solve the equation $6^t=50$ for t.

$\ln 6^t=\ln 50$ $\quad t\ln 6=\ln 50$ $\quad t=\frac{\ln 50}{\ln 6}\approx 2.183$

37. Solve the equation $\log_m 64=6$ for m.

$m^6=64$ $\quad m=2$ $\quad m\neq -2$ because bases cannot be negative

38. Solve the equation $\log k=-2$ for k.

$\log_{10} k=-2$ $\quad 10^{-2}=k$ $\quad k=\frac{1}{100}$

39. Solve the equation $\log_2 t+\log_2(t+14)=5$ for t.

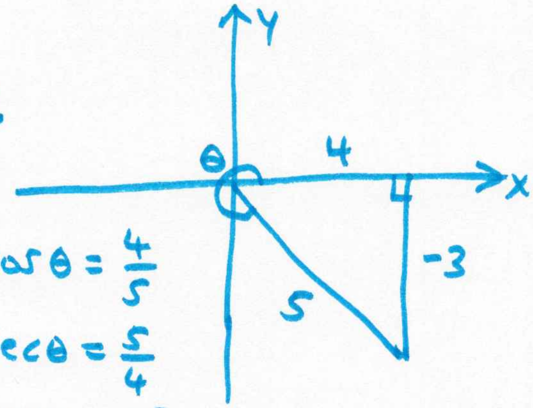
$\log_2(t^2+14t)=5$ $\quad 2^5=t^2+14t$ $\quad 0=t^2+14t-32$ $\quad 0=(t+16)(t-2)$

$\{2\}$

40. Solve the inequality $\ln x>0$ for x. $x>1$

41. True or False: $\log_n x-\log_n y=\frac{\log_n x}{\log_n y}=\log_n\left(\frac{x}{y}\right)$

42. True or False: $\log_a x=\frac{\ln x}{\ln a}$



47.

$\cos\theta = \frac{4}{5}$

$\sec\theta = \frac{5}{4}$

$\csc\theta = \frac{-5}{3}$

$\tan\theta = \frac{-3}{4}$

$\cot\theta = \frac{-4}{3}$

43. Evaluate or simplify the expression $a^{\log_a 7}$ $= 7$

44. Evaluate or simplify the expression $\ln e^4$. $= 4$

45. True or False: $g(x) = \sin x$ is an even function.

46. True or False: $k(x) = \cos x$ is an odd function.

47. Find all the trigonometric values of θ if $\sin\theta = \frac{-3}{5}$ and $\tan\theta < 0$

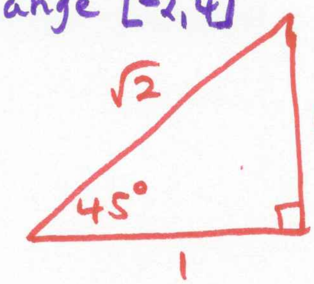
48. Determine the amplitude, period, phase shift, vertical shift and range of

$2(x + \frac{\pi}{2})$

$f(x) = -3\sin(2x + \pi) + 1$ Vertical shift is 1 amplitude is 3

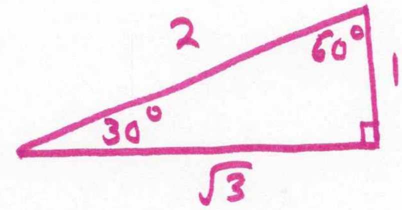
Period is $\frac{2\pi}{2} = \pi$ Phase shift left $\frac{\pi}{2}$ Range $[-2, 4]$

49. Determine the value of each trigonometric expression:



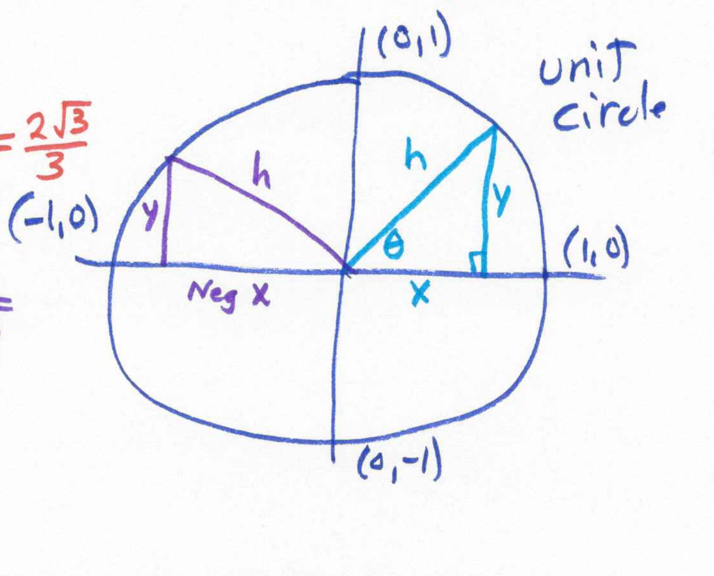
a. $\sin\frac{\pi}{4}$ $= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b. $\cos\frac{\pi}{3}$ $= \frac{1}{2}$



c. $\tan\frac{\pi}{2}$ $= \frac{1}{0} =$ undefined

d. $\cos 0$ $= 1$



$\sin 60° = \frac{\sqrt{3}}{2}$

e. $\csc\frac{2\pi}{3}$ $= \frac{1}{\sin 120°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

f. $\sec\frac{5\pi}{6}$ $= \frac{1}{\cos 150°} = \frac{1}{-\frac{\sqrt{3}}{2}} = \frac{-2}{\sqrt{3}}$

g. $\cot\frac{7\pi}{6} = \frac{1}{\tan\left(\frac{7\pi}{6}\right)} = \frac{1}{\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

h. $\sin\frac{3\pi}{2} = -1$

i. $\cos\pi = -1$

j. $\sec\frac{5\pi}{4} = \frac{1}{\cos\left(\frac{5\pi}{4}\right)} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

k. $\cot\frac{5\pi}{3} = \frac{1}{\tan\frac{5\pi}{3}} = \frac{1}{-\sqrt{3}} = \frac{-\sqrt{3}}{3}$

50. Determine the value of each inverse trigonometric expression:

a. $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

b. $\cos^{-1}(1) = 0$

c. $\tan^{-1}(1) = \frac{\pi}{4}$

d. $\csc^{-1}(2) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

e. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

f. $\cot^{-1}(-1) = \pi$